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Test of spin symmetry in anti-nucleon spectra

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Abstract. The spin symmetry in antinucleon spectra of a nucleus is tested by investigating the relations between the Dirac wave functions of the spin doublets and examining these relations in realistic nuclei within the relativistic mean-field model. In addition to the fact that the dominant components of the Dirac spinors of the spin doublet are nearly identical to each other, there is a differential relation between the smaller components which is found to be almost exactly fulfilled.

PACS. 14.20.-c Baryons (including antiparticles) – 21.10.Hw Spin, parity, and isobaric spin – 21.10.Pc Single-particle levels and strength functions – 03.65.Ge Solutions of wave equations: bound states

1 Introduction

The relativistic mean-field (RMF) [1] model has been very successful in describing properties of nuclear matter and finite nuclei [2–8]. In the relativistic description of the nucleus, the nucleons are treated as Dirac particles interacting with each other via exchanges of various mesons. In the RMF model, although the binding energy of each single nucleon is small (a few tens of MeV or less) compared to the nucleon mass, both the attractive scalar and repulsive vector potentials are very strong. This feature gives rise to many important results. One of them is that the long-standing problem, the origin of the pseudospin symmetry in nuclei was neatly solved [9–13].

The concept of pseudospin [14,15] is based on the experimental observation of the quasidegeneracy between two single-particle orbitals with nonrelativistic quantum numbers $\left(n_r, l, j = l + \frac{1}{2}\right)$ and $\left(n_r - 1, l + 2, j = l + \frac{3}{2}\right)$, where n_r, l and j are the radial quantum number, the orbital and total angular momenta, respectively. By introducing the "pseudo" orbital angular momentum $\tilde{l} = l + 1$ and "pseudo" spin $\tilde{s} = s = \frac{1}{2}$, these degenerate states can be expressed conveniently in terms of the pseudospin doublet with $j = \tilde{l} \pm \tilde{s}$.

In ref. [10], it is clearly shown that the pseudo quantum numbers of a particle state with positive energy are nothing but the quantum numbers of the lower component and the pseudospin symmetry is a symmetry of the lower component of the Dirac spinor. In the Dirac equation of the nucleon, when the scalar potential $S(\boldsymbol{r})$ and the vector potential $V(\boldsymbol{r})$ are equal in amplitudes but opposite in sign, *i.e.*, $S(\boldsymbol{r}) + V(\boldsymbol{r}) = 0$, or more generally, $\mathrm{d}[S(\boldsymbol{r}) + V(\boldsymbol{r})]/\mathrm{d}r = 0$, there is an exact pseudospin symmetry in single-particle spectra [10–12,16–18]. Under these conditions, there should be some special relations between the four components of the Dirac wave functions of the pseudospin doublets which have been used to test the pseudospin symmetry in realistic nuclei with spherical or axially deformed shapes [19–22]. The readers are referred to ref. [23] for a recent of review on relativistic (spin and pseudospin) symmetries in atomic nuclei.

The lower component of the Dirac spinor corresponds to the antiparticle degree of freedom [24], and the spin symmetry in the antinucleon spectra was proposed and justified in real nuclei [19,25]. It was found that not only the energies of a pair of spin partners are nearly degenerate but also the larger components of their Dirac wave functions are almost identical [25]. One also expects some other special relations between the four components of the Dirac wave functions of a spin doublet in the antinucleon spectra. The differential relations between the eigenfunctions of spin partners in nucleon spectra have been tested [26]. In the present work, we shall investigate these relations and test the spin symmetry in antinucleon spectra in realistic nuclei.

The relations between the four components of the Dirac wave functions of a spin doublet are derived in the next section. In sect. 3, the RMF calculations in some

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spherical nuclei are performed in order to test these relations. A brief summary is given in sect. 4.

2 Spin symmetry conditions on the Dirac spinors

The Dirac equation for nucleons in the relativistic mean-field model reads

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p} + V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r}))]\,\psi(\boldsymbol{r}) = \varepsilon\psi(\boldsymbol{r}). \tag{1}$$

Considering a spherical system, the radial quantum number n_r, the total angular momentum j, its projection on the z-axis m, and κ are conserved. $\kappa = (-)^{j+l+1/2}(j+1/2)$ is the eigenvalue of $\hat{\kappa} = -\beta\left(\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{l}} + 1\right)$ which characterizes the spin-orbit operator. The quantum number κ and the radial quantum number n_r can be used to label an orbit with $|2\kappa|$ degeneracy. Then the Dirac wave function for a nucleon $\psi(\boldsymbol{r})$ is given by

$$\psi(\boldsymbol{r}) = \frac{1}{r}\begin{pmatrix} iG_{n\kappa}(r)Y^{l}_{jm}(\theta,\phi,s) \\ -F_{\tilde{n}\kappa}(r)Y^{\tilde{l}}_{jm}(\theta,\phi,s)\end{pmatrix} \tag{2}$$

with

$$Y^{l}_{jm}(\theta,\phi,s) = \sum_{m_l,m_s}\left\langle l m_l \frac{1}{2} m_s \middle| jm\right\rangle Y_{lm_l}(\theta,\phi)\chi_{m_s}(s),$$

$G_{n\kappa}(r)/r$ and $F_{\tilde{n}\kappa}(r)/r$ are the radial wave functions, and $\tilde{l} = l - \mathrm{sign}(\kappa)$ the orbital angular momentum of the lower component.

The charge conjugation leaves the scalar potential invariant while it changes the sign of the vector potential. Therefore, the Dirac equation for antinucleons reads

$$[\boldsymbol{\alpha}\cdot\boldsymbol{p} - V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r}))]\,\psi(\boldsymbol{r}) = \varepsilon\psi(\boldsymbol{r}) \tag{3}$$

and the Dirac wave function for an antinucleon is given by the charge conjugation,

$$\psi(\boldsymbol{r}) = \frac{1}{r}\begin{pmatrix} -F_{\tilde{n}\tilde{\kappa}}(r)Y^{\tilde{l}}_{jm}(\theta,\phi,s) \\ iG_{n\tilde{\kappa}}(r)Y^{l}_{jm}(\theta,\phi,s)\end{pmatrix} \tag{4}$$

with $\tilde{\kappa} = (-)^{j+\tilde{l}+1/2}(j+1/2)$, $n = \tilde{n}+1$ for $\tilde{\kappa} > 0$ and $n = \tilde{n}$ for $\tilde{\kappa} < 0$ [25,27]. We adopt the convention that the quantum numbers of a state follow those of its dominant component. Thus, an antinucleon state is labelled by $\{\tilde{n},\tilde{l},\tilde{\kappa},m\}$, its pseudo quantum numbers are $\{n,l,\kappa,m\}$, while a particle state is labelled by $\{n,l,\kappa,m\}$ with pseudo quantum numbers $\{\tilde{n},\tilde{l},\tilde{\kappa},m\}$.

The Dirac equation (3) is reduced to two coupled radially differential equations,

$$\left[\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\tilde{\kappa}}{r}\right]F(r) = [-\varepsilon - M + V_+(r)]\,G(r), \tag{5}$$

$$\left[\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\tilde{\kappa}}{r}\right]G(r) = [+\varepsilon - M - V_-(r)]\,F(r), \tag{6}$$

where $V_\pm(r) = V(r) \pm S(r)$. A Schrödinger-like equation for the upper component $F(r)$ is obtained as

$$\left[-\frac{1}{2M_-}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{1}{2M_-}\frac{\mathrm{d}V_+}{\mathrm{d}r}\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\tilde{l}\left(\tilde{l}+1\right)}{r^2}\right)\right]F(r) + \left[\frac{1}{4M_-^2}\frac{\tilde{\kappa}}{r}\frac{\mathrm{d}V_+}{\mathrm{d}r} + M - V_-\right]F(r) = \varepsilon F(r), \tag{7}$$

where $M_\pm = M \pm \varepsilon \mp V_\mp$.

When $\mathrm{d}V_+/\mathrm{d}r = 0$, there is an exact spin symmetry in the antiparticle spectra [25]. The dominant components $F(r)$ of a pair of spin partners are also identical. In real nuclei, since $\mathrm{d}V_+/\mathrm{d}r \neq 0$, this symmetry is broken. Because the term with $\mathrm{d}V_+/\mathrm{d}r$ is fairly small, the spin symmetry in antiparticle spectra is approximately conserved.

Rewriting eq. (6) for a spin doublet as

$$F_{\tilde{l}\pm\frac{1}{2}}(r) = \frac{1}{\varepsilon - M - V_-(r)}\left[\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\tilde{\kappa}}{r}\right]G_{\tilde{l}\pm\frac{1}{2}}(r), \tag{8}$$

and using the relations $\varepsilon_{\tilde{l}+\frac{1}{2}} = \varepsilon_{\tilde{l}-\frac{1}{2}}$ and

$$F_{\tilde{l}+\frac{1}{2}}(r) = F_{\tilde{l}-\frac{1}{2}}(r), \tag{9}$$

when $\mathrm{d}V_+/\mathrm{d}r = 0$, one arrives at the differential relation between the lower components of the Dirac wave functions,

$$\left(\frac{\mathrm{d}}{\mathrm{d}r} + \frac{\tilde{l}+1}{r}\right)G_{\tilde{l}+1/2}(r) = \left(\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\tilde{l}}{r}\right)G_{\tilde{l}-1/2}(r). \tag{10}$$

Here the radial quantum number $\tilde{n}_r$ is omitted for brevity. We note that relation (10) is similar to the differential relation between the two upper components of a pseudospin doublet (eq. (40) given in ref. [21]). We will examine to what extent the relations given in eqs. (9) and (10) are fulfilled in realistic nuclei which would tell us how well the spin symmetry in antinucleon spectra is conserved.

3 Test of the spin symmetry relations in realistic nuclei

We carried out relativistic mean-field calculations for realistic nuclei. Under the mean-field and no-sea approximations, the equations of motion for nucleons, mesons and photon are solved in coordinate space. The parameter set NL3 [28] is used in our calculation. With the vector and scalar potentials obtained by solving the RMF equations self-consistently, eq. (3) is solved and we get the eigenenergies and eigenfunctions for antinucleons. Good spin symmetry is found both in the antiproton and in the antineutron spectra. In the following the results for antineutrons will be given.

First the antineutron states for the doubly magic nucleus ^{40}Ca are shown in fig. 1. For each pair of the spin doublets, the left level is with $\tilde{l} - 1/2$ and the right one with $\tilde{l} + 1/2$.

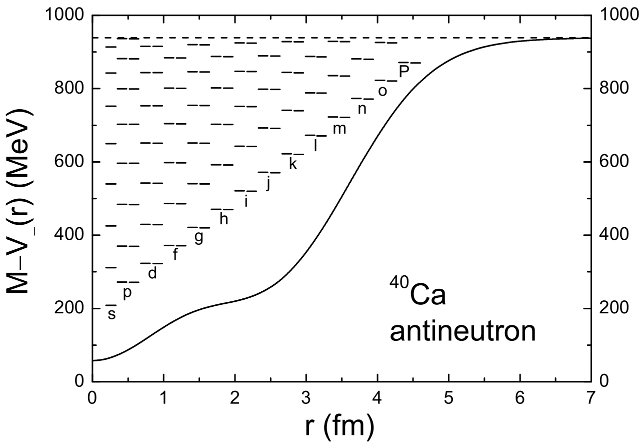
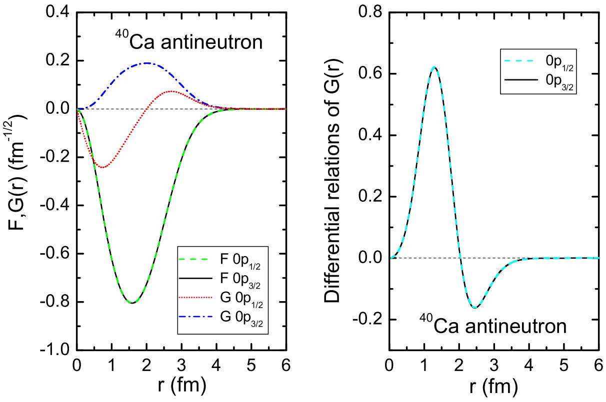


Fig. 1. Antineutron potential and spectrum of ^{40}Ca. For each pair of the spin doublets, the left level is with $\tilde{l}-1/2$ and the right one with $\tilde{l}+1/2$.

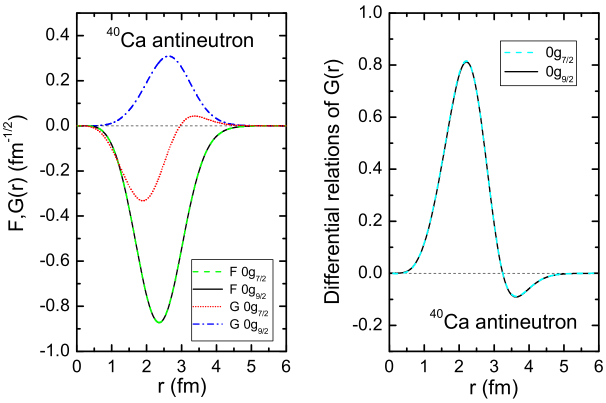
The radial wave functions for the antineutron spin doublet $0p_{1/2}$ and $0p_{3/2}$ in ^{40}Ca are shown in the left panel of fig. 2(a). This is the spin doublet with the lowest orbital angular momentum $\tilde{l}=1$. The energies of these two states are 271.91 and 271.55 MeV, respectively. We can see that the upper components $F(r)$ of the eigenfunctions for the spin doublet are almost exactly identical with each other due to the good spin symmetry. But the lower component $G(r)$ of the wave function of an antineutron state deviates dramatically from that of its spin partner. In the right panel of fig. 2(a), the differential relation of the lower components given in eq. (10) is presented. One finds that this differential relation is satisfied remarkably well which gives a further support to the spin symmetry in the antinucleon spectra in nuclei.

With orbital angular momentum $\tilde{l}$ increasing, the spin-orbit splitting becomes larger [25]. Next we examine relations (9) and (10) for spin doublets with larger $\tilde{l}$. In figs. 2(b) and (c), the results of the spin doublets $0g_{7/2,9/2}$ and $0j_{13/2,15/2}$ with $\tilde{l}=4$ and 7 are presented, respectively. The energies of $0g_{7/2,9/2}$ are 421.19 and 420.38 MeV and those of $0j_{13/2,15/2}$ are 563.46 and 562.29 MeV. The spin-orbit splitting increases only slightly with $\tilde{l}$ increasing and the spin symmetry conditions (9) and (10) are still met well for spin doublets with larger $\tilde{l}$.

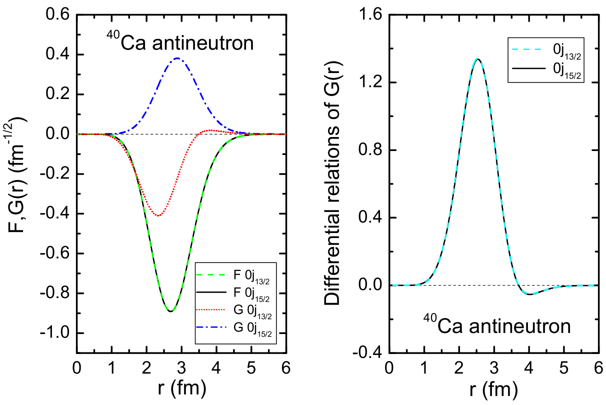
One is also interested in how the spin symmetry develops with the radial quantum number $\tilde{n}_r$ increasing. For this purpose we examine the wave function relations (9) and (10) for spin doublets with larger $\tilde{n}_r$. The wave function relations of the spin doublets $3p_{1/2,3/2}$ and $6p_{1/2,3/2}$ with $\tilde{n}_r=3$ and 6 are shown in figs. 3(a) and (b), respectively. The energies of $3p_{1/2,3/2}$ are 596.60 and 596.27 MeV and those of $6p_{1/2,3/2}$ are 881.73 and 881.45 MeV. The spin-orbit splitting decreases when the radial quantum number increases. It can be seen from fig. 3 that the relations of the wave functions for the spin doublets (9) and (10) are fulfilled for spin doublets with large $\tilde{n}_r$ as well.



(a) The antineutron spin doublet $0p_{1/2}$ (ε = 271.91 MeV) and $0p_{3/2}$ (ε = 271.55 MeV) in ^{40}Ca.

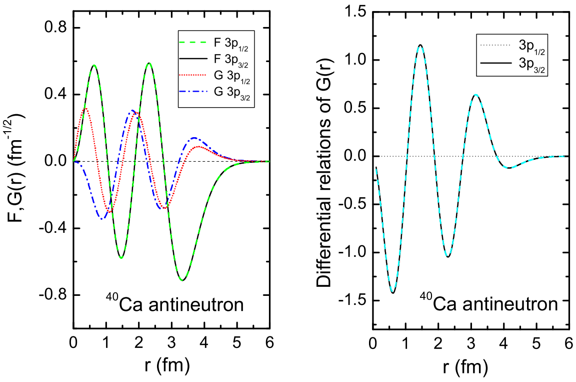


(b) The antineutron spin doublet $0g_{7/2}$ (ε = 421.19 MeV) and $0g_{9/2}$ (ε = 420.38 MeV) in ^{40}Ca.

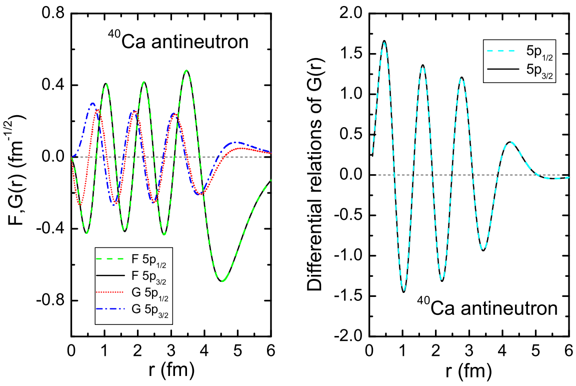


(c) The antineutron spin doublet $0j_{13/2}$ (ε = 563.46 MeV) and $0j_{15/2}$ (ε = 562.29 MeV) in ^{40}Ca.

Fig. 2. (Color online) Radial wave functions (left panel) and the differential relation (eq. (10)) of the lower components (right panel) of the antineutron spin doublets with different orbital angular momentum $\tilde{l}$.



(a) The antineutron spin doublet $3p_{1/2}$ (ε = 596.60 MeV) and $3p_{3/2}$ ($\varepsilon = 596.27$ MeV) in ^{40}Ca.

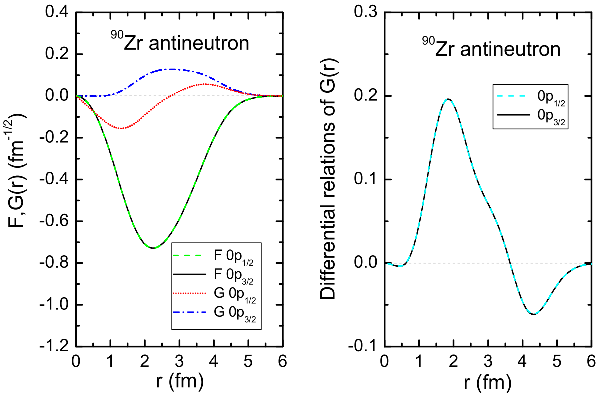


(b) The antineutron spin doublet $6p_{1/2}$ (ε = 881.73 MeV) and $6p_{3/2}$ ($\varepsilon = 881.45$ MeV) in ^{40}Ca.

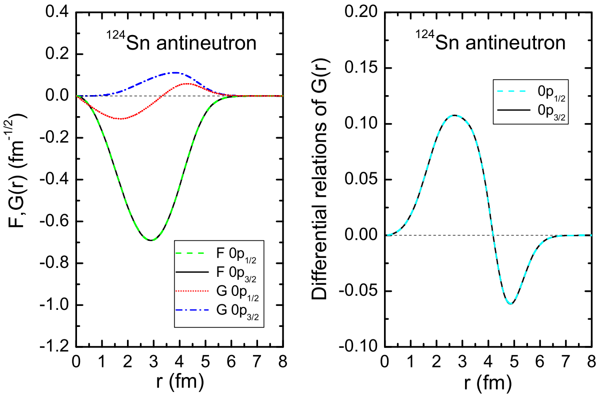
Fig. 3. (Color online) Radial wave functions (left panel) and the differential relation (eq. (10)) of the lower components (right panel) of the antineutron spin doublets with different radial quantum number $\tilde{n}_r$ and $\tilde{l} = 1$.

To test the validity of spin symmetry relations (9) and (10) throughout the periodic table, we carried out calculations for many other nuclei. We choose several of them as examples, namely, ^{90}Zr, ^{124}Sn, and ^{208}Pb. The results for antineutron spin doublets $0p_{1/2}$ and $0p_{3/2}$ in these nuclei are shown in fig. 4. The energies of these antineutron states are 252.27 and 252.23 MeV for ^{90}Zr, 261.60 and 261.59 MeV for ^{124}Sn, and 273.74 and 273.74 MeV for ^{208}Pb. In heavier nuclei the spin symmetry is more conserved which is inferred from the smaller spin orbit splittings. From fig. 4 one can find that the relations between the radial wave functions of a spin doublet in antineutron spectra are met well in these nuclei. Similar conclusions hold for other nuclei and results will not be detailed here.

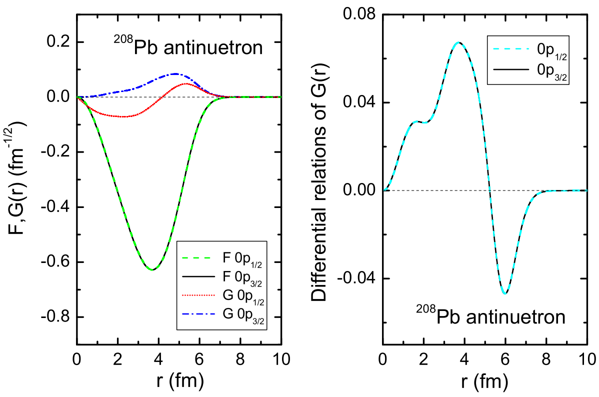
We notice that in the present calculations the coupling constants of the antinucleon and the nucleon are related simply by the G-parity transformation [29]. However, there are several effects which cause deviations of the



(a) The antineutron spin doublet $0p_{1/2}$ (ε = 252.27 MeV) and $0p_{3/2}$ ($\varepsilon = 252.23$ MeV) in ^{90}Zr.



(b) The antineutron spin doublet $0p_{1/2}$ (ε = 261.60 MeV) and $0p_{3/2}$ ($\varepsilon = 261.59$ MeV) in ^{124}Sn.



(c) The antineutron spin doublet $0p_{1/2}$ (ε = 273.74 MeV) and $0p_{3/2}$ ($\varepsilon = 273.74$ MeV) in ^{208}Pb.

Fig. 4. (Color online) Radial wave functions (left panel) and the differential relation (eq. (10)) of the lower components (right panel) of the antineutron spin doublets $0p_{1/2,3/2}$ for ^{90}Zr, ^{124}Sn, and ^{208}Pb.

coupling constants of the antinucleons from the *G*-parity values, *e.g.*, the inclusion of the Fock terms or the annihilation channels of the antinucleons [30]. One is reminded that the present RMF calculations are carried out under the "no-sea" approximation. The contribution of the Dirac sea results in smaller potentials for antinucleons thus changing the antinucleon spectra [31]. Furthermore, although "the polarization of the nucleus by the atomic antiproton is negligible" [32], the polarization effects caused by the antinucleon inside a nucleus are considerably large and change both the vector and the scalar potentials [33]. These effects would make the spin symmetry discussed here be broken more. But the qualitative picture will not change. The annihilation probability of the antinucleon in the nucleus might be very large and could also change the simple picture of the spin symmetry presented here.

4 Summary

In summary, we test the spin symmetry in antinucleon spectra in nuclei by investigating the spin symmetry conditions on the four components of Dirac spinors of spin doublets. The relativistic mean-field calculations are carried out and the spin symmetry conditions are examined in nuclei ^{40}Ca, ^{90}Zr, ^{124}Sn, and ^{208}Pb. In addition to the fact that the dominant components of the Dirac spinors of the spin doublets are almost identical (eq. (9)), there is a differential relation between the lower components (eq. (10)) which is met very well.

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